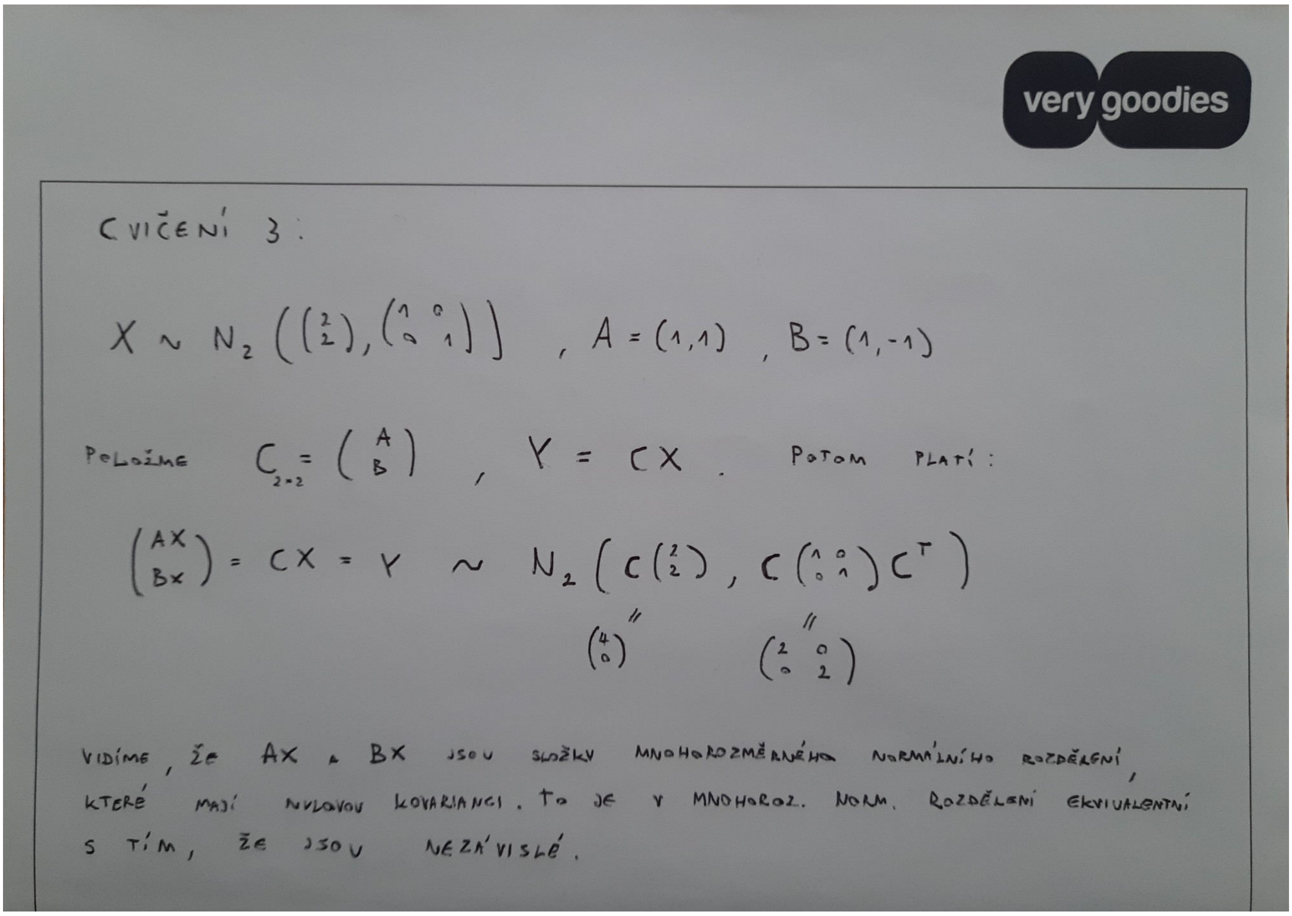
3. Assignment

- Theoretical task – third exercise



Cvičení 3:

$$X \sim N_2\left(\begin{pmatrix}2\\2\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\right), \quad A = (1,1), \quad B = (1,-1)$$

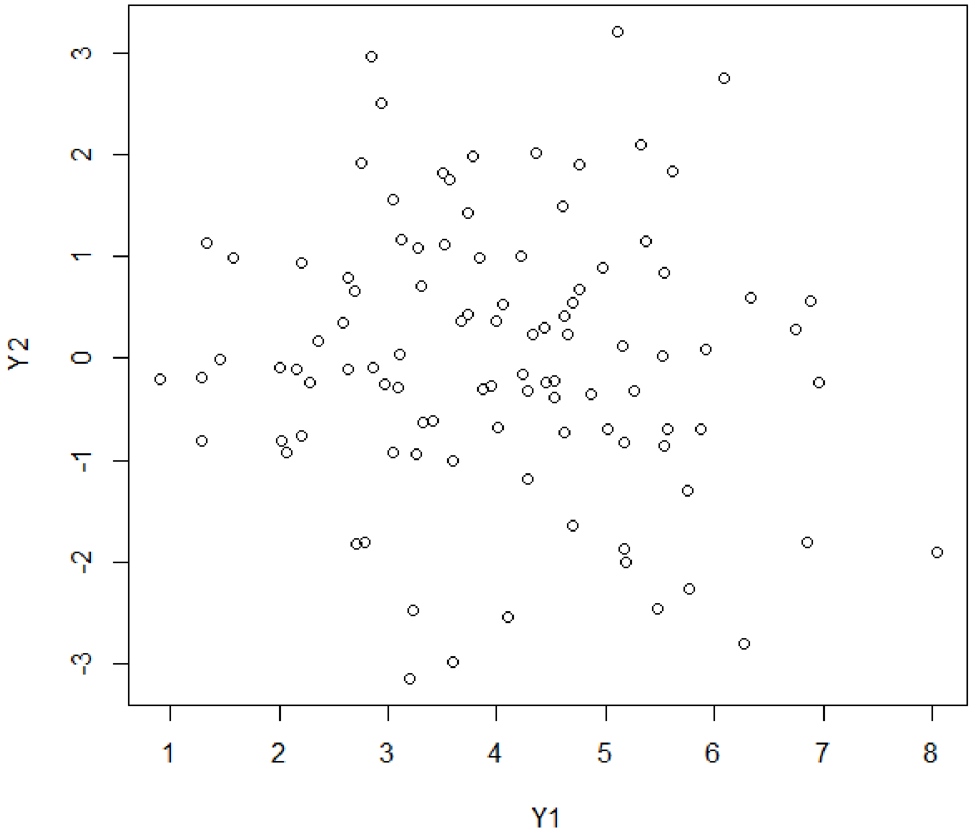
Položme $\underset{2\times 2}{C} = \begin{pmatrix}A\\B\end{pmatrix}$, $Y = CX$. Potom platí:

$$\begin{pmatrix}AX\\BX\end{pmatrix} = CX = Y \sim N_2\left(C\begin{pmatrix}2\\2\end{pmatrix}, C\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}C^T\right)$$

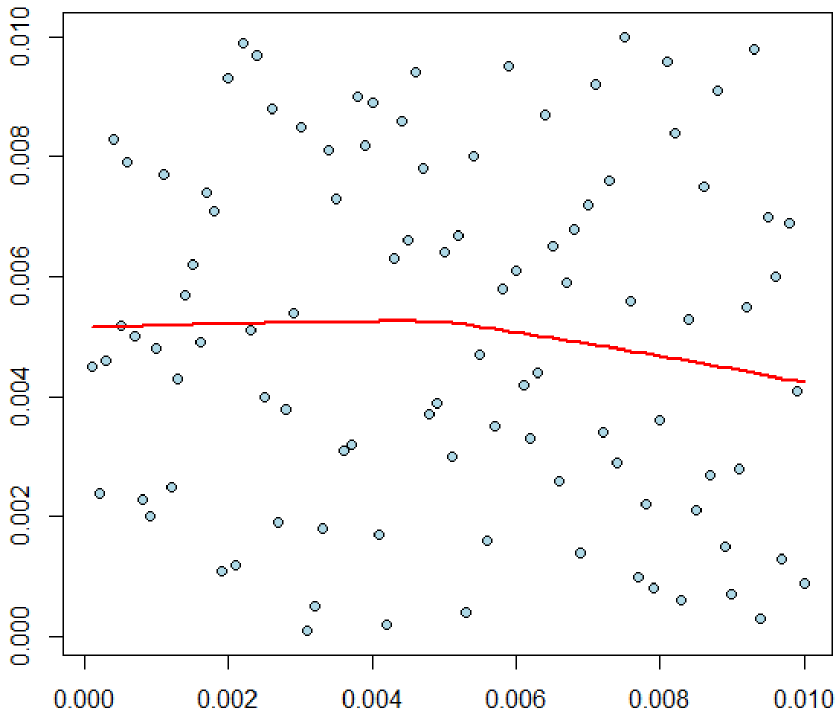
$$C\begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix}, \quad C\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}C^T = \begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$$

Vidíme, že AX a BX jsou složky mnohorozměrného normálního rozdělení, které mají nulovou kovarianci. To je v mnohoroz. norm. rozdělení ekvivalentní s tím, že jsou nezávislé.

To visualize that random variables and are independent, we can take a look at the scatterplot of 100 random vectors, which were generated from the distribution of random vector :



For a closer look at the independence, scatterplot of the random variables as arguments of their univariate empirical distribution functions is also present, together with the non-parametric estimator of the effect. Both presented graphs show the independence well.



- **Second task – Marginal does not imply joint distribution**

CONSIDER 2 IID R. VARIABLES X, Y, BOTH FOLLOWING $N(0,1)$ DIST.

LET $Z := \text{sgn}(X) \cdot |Y|$. IT HOLDS:

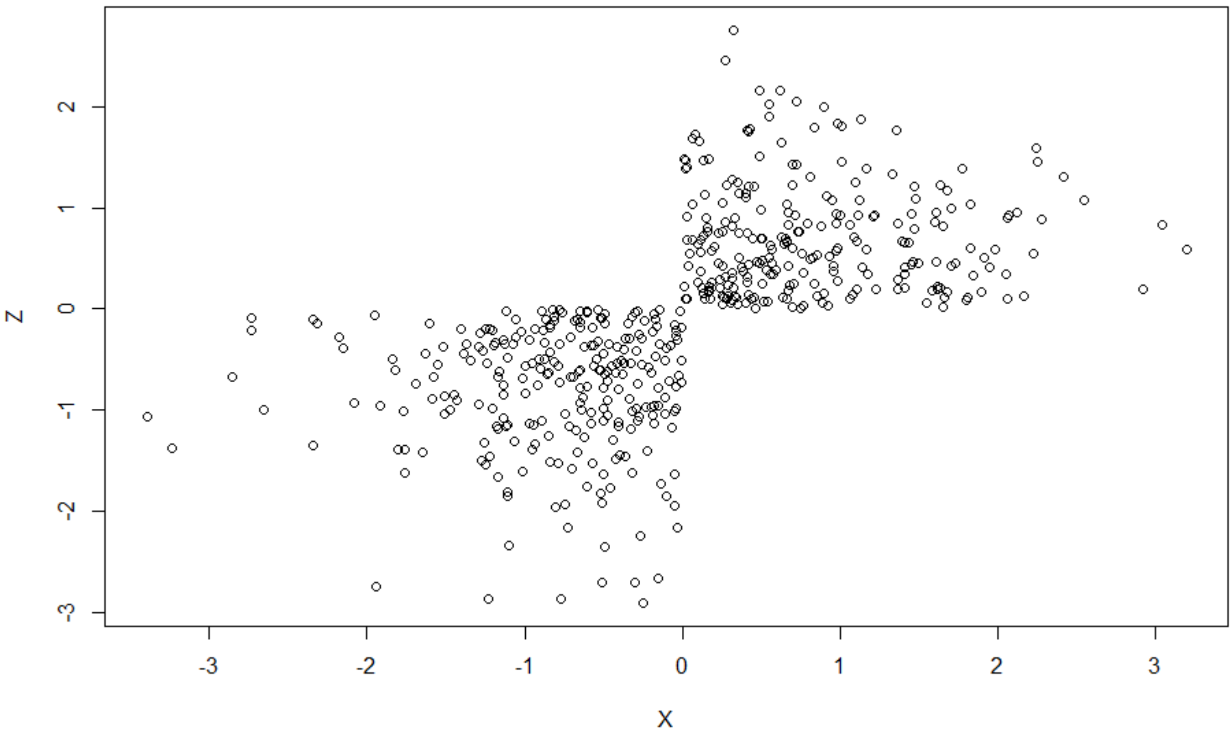
$$P(Z \le z) = P(\text{sgn}(X) \cdot |Y| \le z) = P(\text{sgn}(X) = -1, -|Y| \le z)$$

$$+ P(\text{sgn}(X) = 1, |Y| \le z) \overset{\text{indep.}}{=} \frac{1}{2}\Big(P(-|Y| \le z) + P(|Y| \le z)\Big)$$

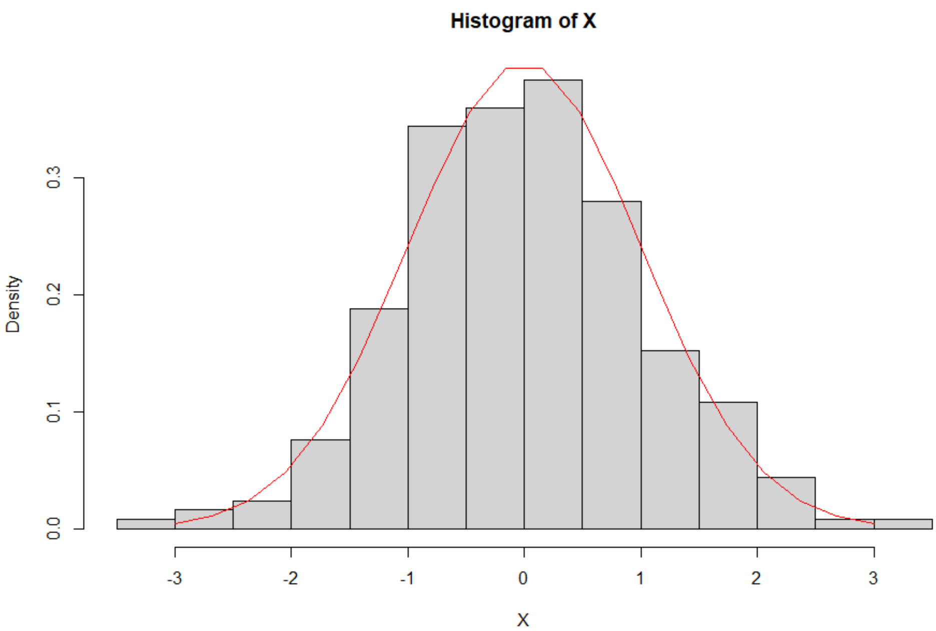
$$= \begin{cases} \frac{1}{2}\big(1 + \Phi(z) - \Phi(-z)\big) \overset{\text{symm.}}{=} \Phi(z) & z > 0 \\ \frac{1}{2}\big(P(Y \le z) + P(Y \ge -z) + 0\big) = \frac{1}{2}\big(\Phi(z) + 1 - \Phi(-z)\big) = \Phi(z) & z < 0 \end{cases}$$

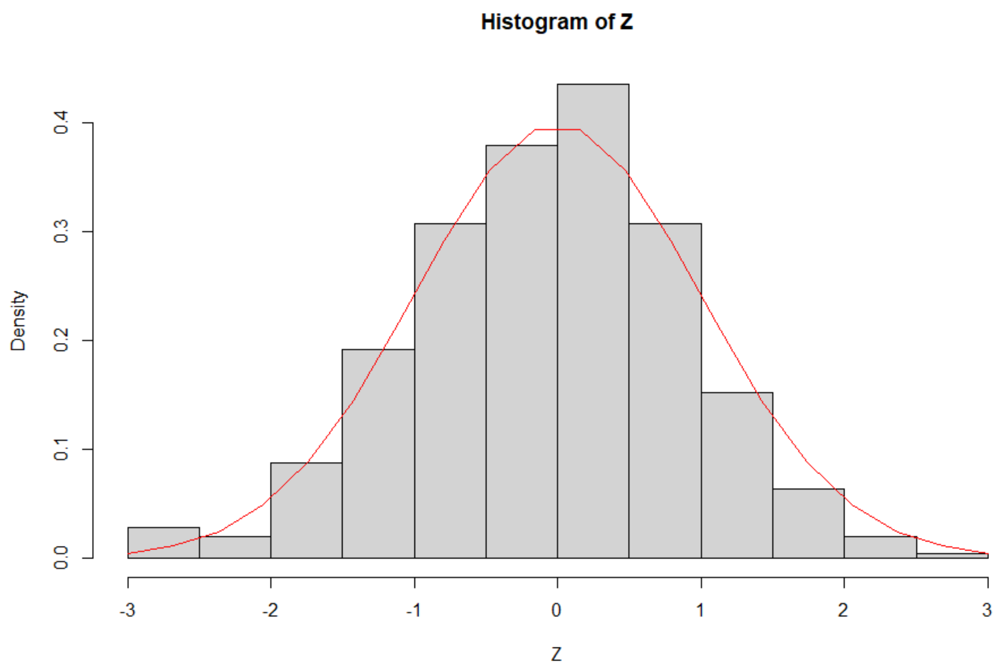
BOTH X AND Z FOLLOW $N(0,1)$ DISTRIBUTION, BUT $\{(x,z)^T ; f_{X,Z}((x,z)^T) > 0\} \neq \mathbb{R}^2$ (SUPPORT OF THE JOINT DIST. ISN'T $\mathbb{R}^2$), THEREFORE $(X,Z)^T$ ISN'T MULTIVARIATE NORMAL.

To prove that the presented joint distribution does not follow multivariate normal distribution, I used the knowledge of the support. The situation is even clearer, if we generate 500 observations from the corresponding joint distribution, and take a look at the scatterplot:



Clearly, the joint distribution is not multivariate normal. We should also make sure that the marginal distributions truly follow the normal distribution. Histograms of both random variables, together with their theoretical densities, can be seen below.




Histogram of Z
Density
0.0
0.1
0.2
0.3
0.4
-3
-2
-1
0
1
2
3
Z