

Multivariate analysis - hw5

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1 Estimating type I error

Our first task is to compare empirical type I errors for 2 different tests with the same null hypothesis. More specifically, we have a random sample from bivariate normal distribution with parameters $\mu = (1, 2)^T$ and $\Sigma = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 4 \end{pmatrix}$.

We want to test the null hypothesis that for $a = (2, -1)^T$, it holds that $\mu^T a = 0$ (which clearly holds for our data). To test the hypothesis, we will use 2 different tests, one in which we know the variance matrix, and one in which we doesn't. We will compare the empirical type I errors of the tests for different sample sizes. To obtain the estimates, we will simulate 500 tests of both types for every considered sample size. The results can be seen in figure 1.

We see that for both tests the empirical type I errors get closer to the set significance level as the sample size rises. From the graph, it's hard to say which test is generally better.

2 Estimating the test power

Another important factor for choosing a test is its power. We used the same procedure that we used in section 1, only this time, the data was generated with expected value $\mu = (1.1, 2)^T$. We see that the null hypothesis used before doesn't hold anymore, so now instead of type I error, we are interested in estimating the power of the tests. The corresponding graph can be seen in figure 2. To see how much the violation of the null hypothesis affects the test power, we also created the same graph for case, in which $\mu = (1.4, 2)^T$ in figure 3.

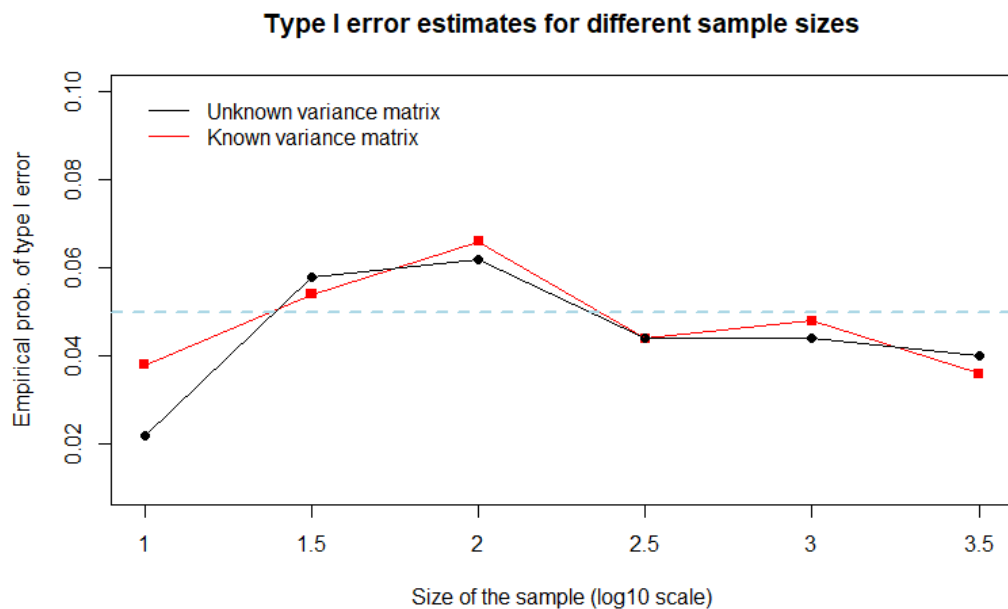


Figure 1: Comparison of empirical type one errors for the 2 tests. Tests were conducted on 5% significance level, which is displayed by the blue dashed line.

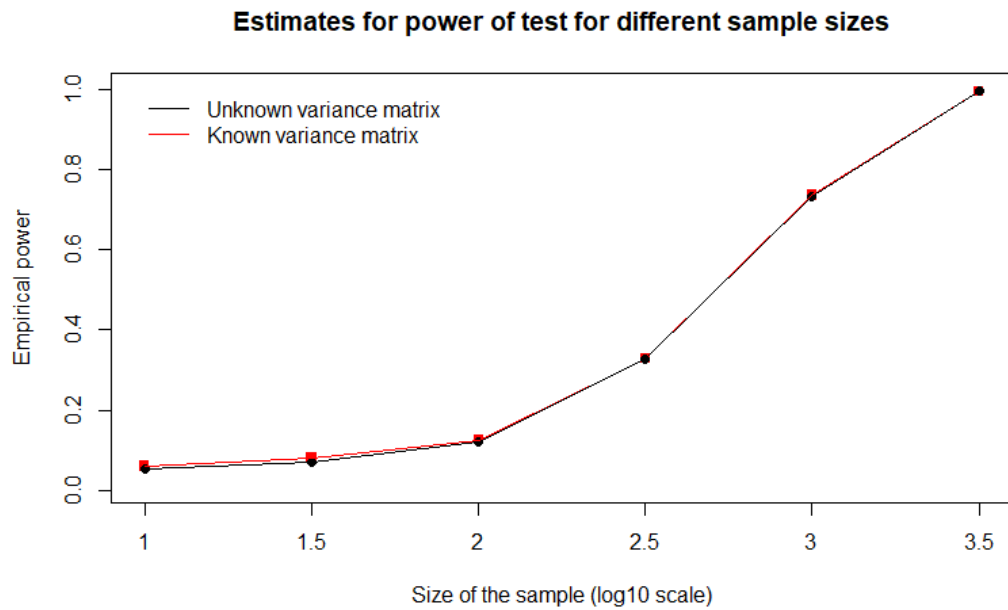


Figure 2: Comparison of empirical power after slightly changing the expected value of distribution, from which the data was generated. Both tests seem to have almost the same power.

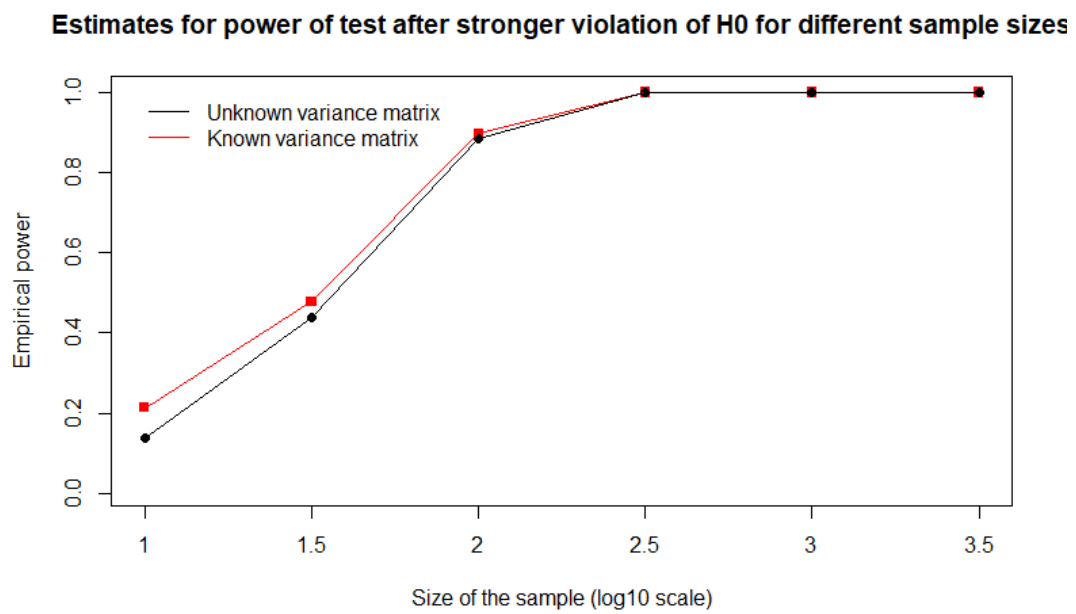


Figure 3: Comparison of empirical power after changing the expected value by a bigger margin. We see that estimates of power are larger than before, with the test with known variance converging faster to 1.