5. Assignment NMST 539

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1 First type error

In this assignment we simulated a random sample from the bivariate normal distribution $N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} = (1,2)^T$ and variance matrix $\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0,5 \\ 0,5 & 2 \end{pmatrix}$. For all simulation we run it 1000 times.

we will testing the null hypothesis is $H_0 : \mathbb{A}\boldsymbol{\mu} = 0$ agaisnt the alternative $H_1 : \mathbb{A}\boldsymbol{\mu} \neq 0$, where $\mathbb{A} = (2, -1)^T$.

Firstly, we will assume that the variance matrix $\boldsymbol{\Sigma}$ is known. Under the null hypothesis:

$$n\left(\mathbb{A}\overline{\mathbf{X}}_n\right)^T\left(\mathbb{A}\boldsymbol{\Sigma}\mathbb{A}^T\right)^{-1}\left(\mathbb{A}\overline{\mathbf{X}}_n\right) \sim \chi^2_1$$

holds. The first type error for the known variance matrix can be seen in the figure 1.

Secondly, we will not assume that the variance matrix is known. We have to use the sample variance-covariance matrix. under the null hypothesis, the distribution below holds.

$$(n-1)\left(\mathbb{A}\overline{\mathbf{X}}_n\right)^T\left(\mathbb{A}\mathbf{S}\mathbb{A}^T\right)^{-1}\left(\mathbb{A}\overline{\mathbf{X}}_n\right) \sim T^2_{1,n-1}.$$

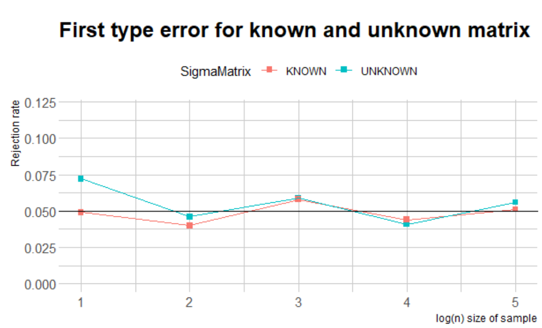

Figure 1: First type error.

The first type error for the unknown variance matrix can be seen in the figure 1. We can see that for a higher size random sample, the rejection rate is closer to 0.05, which is the prescribed level.

2 Empirical power of the test

In this part, we will we simulate a random sample from the bivariate normal distribution $N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} = (1.1, 2)^T$ and the variance matrix as defined above. We will test the same null hypothesis. As above, we will use the test for known and unknown variance matrices. The empirical power of the test can be seen in the figure 2.

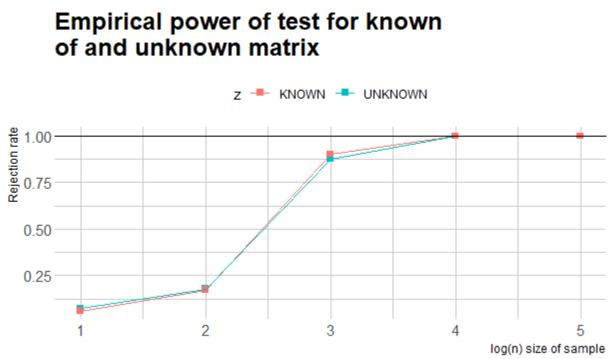

Figure 2: Empirical power of the test.

In both cases, we can see that for a higher sample size, we obtain more precise results. For the sample size greater than 1000, we can observe that the empirical power is almost 100 %.