3. Assignment NMST 539

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1 First part - Exercise 3

In this section, we will go through the solution of the third exercise from the list of theoretical exercises. Consider the random vector $\boldsymbol{X} \sim N_2(\boldsymbol{\mu}, \Sigma)$, for $\boldsymbol{\mu} = (2, 2)$ and Σ is an unit matrix. Consider also matrices $\mathbb{A} = (1, 1)$ and $\mathbb{B}(1, -1)$. Show that the random variables $\mathbb{A}\boldsymbol{X}$ and $\mathbb{B}\boldsymbol{X}$ are independant.

The variable $\boldsymbol{X}$ has multivariate normal distribution. Thus the random variables $\mathbb{A}\boldsymbol{X}$ and $\mathbb{B}\boldsymbol{X}$ have univariate normal distribution with mean values $\mu_{\mathbb{A}\boldsymbol{X}} = 0$ and $\mu_{\mathbb{B}\boldsymbol{X}} = 4$. From the properties of multivariate normal distribution it is enough to prove that the variables $\mathbb{A}\boldsymbol{X}$ and $\mathbb{B}\boldsymbol{X}$ are uncorrelated. That property flows from the following calculation

$$cov(\mathbb{A}\boldsymbol{X}, \mathbb{B}\boldsymbol{X}) = \mathbb{A}var(\boldsymbol{X})\mathbb{B}^T = 1 - 1 = 0.$$

That means that the variables are uncorrelated thus independent. The independence can be also seen in the figure 1.

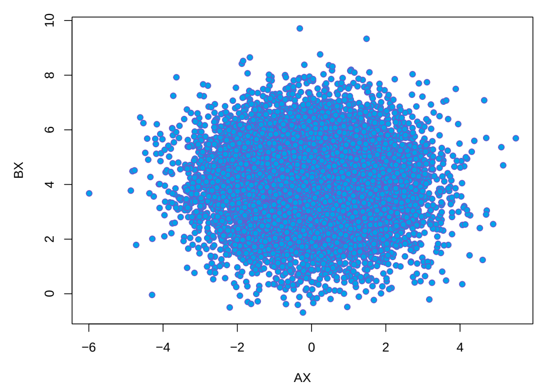

Figure 1

2 Second part

In the second part of the assignment, we will prove that the normal distribution of marginals does not imply the joint multivariate normal distribution.

We will use the standard normal distribution which is restricted only to the first and third quadrants. Let us consider two random variables with standard normal distribution X, Y and their density

$$f_{X,Y}(x,y) = 2f_X(x)f_Y(y) \quad , \text{if } x \geq 0, y \geq 0 \text{ or } x < 0, y < 0$$

The joint distribution is not binormal, because the binormal distribution is defined on the whole $\mathbb{R}^2$. This fact can be also seen in the figure 2 where we generate a random sample from the distribution above. If we focus on marginals, we can see that they are normal. We can use the fact that the density of standard normal distribution is even function and obtain that the marginals have a standard normal distribution.

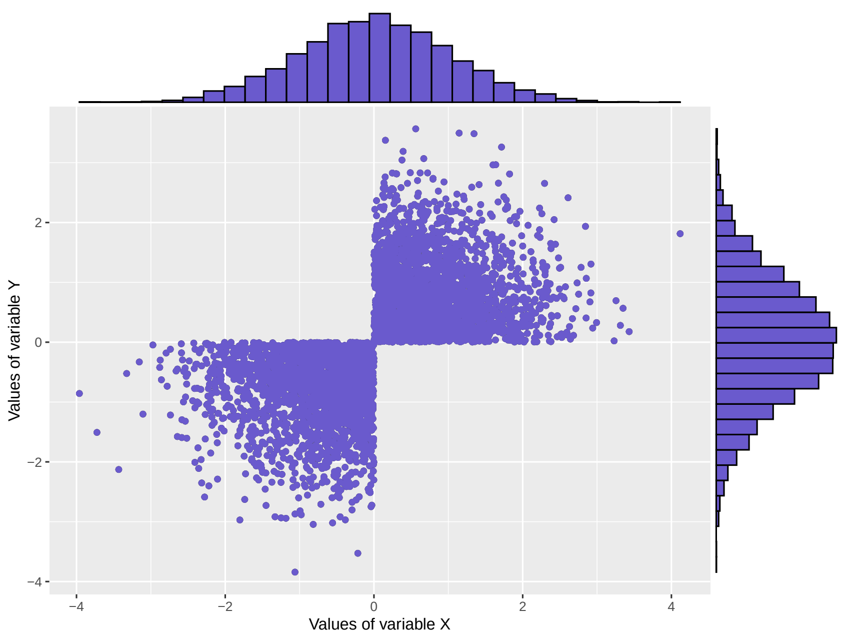

Figure 2