2. Assignment NMST 539

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1 First part

In the first part of the assignment let assume two-dimensional distribution of random vector $(X,Y)^T$ over the set $M = \{(x,y) \in \mathbb{R}^2; 0 < x < y < 1\}$. We will derive the joint density $f_{XY}(x,y)$ and marginal densities $f_X(x)$, $f_Y(y)$.

We know that the density has to be in form $c\ \mathbb{1}_M$, where $c \in \mathbb{R}$ and $\mathbb{1}_M$ is an indicator of the set M. Due to th properties of density, the following equation holds.

$$1 = \int_0^1 \int_0^1 c\ \mathbb{1}_M \, dy \, dx = \int_0^1 \int_x^1 c \, dy \, dx = \frac{c}{2}.$$

From here, we can easily obtain $c = 2$ and the joint density function is $f_{XY}(x,y) = 2\ \mathbb{1}_M$. We can derive the marginal densities by integrating.

$$f_X(x) = \int_x^1 2\ \mathbb{1}_{(0,1)} \, dy = 2y\ \mathbb{1}_{(0,1)}$$

$$f_Y(y) = \int_0^y 2\ \mathbb{1}_{(0,1)} \, dx = 2(1-x)\ \mathbb{1}_{(0,1)}$$

We are also interested in the independence of variables X, Y. It can be seen that the variables are not independent because the following equation does not hold.

$$f_{XY}(x,y) = 2\ \mathbb{1}_M \neq 2(1-x)2y = f_X(x)f_Y(y).$$

Therefore the variables are not independent.

2 Second part

In the second part of the assignment, we will run a simulation to show that the variables follow the density derived in the first part. Our random sample has 1000 observations. Firstly we will compute the sample mean

$$\begin{pmatrix} \overline{X}_n \\ \overline{Y}_n \end{pmatrix} = \begin{pmatrix} 0,3178 \\ 0,6596 \end{pmatrix}$$

and sample covariance matrix

$$cov(X,Y) = \begin{pmatrix} 0,0532 & 0,0263 \\ 0,0263 & 0,0569 \end{pmatrix}.$$

We also run a simulation and obtain figure 1. We can see that joint density follows the density derived in the first part.

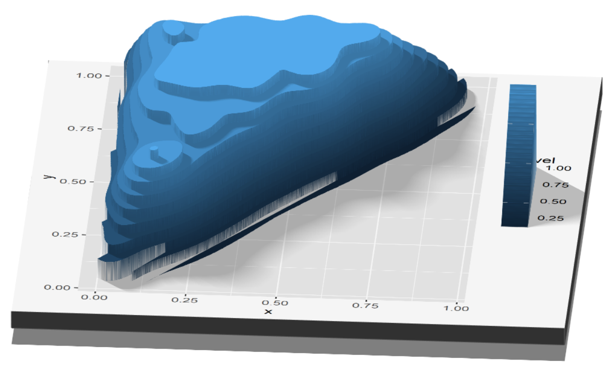

Figure 1

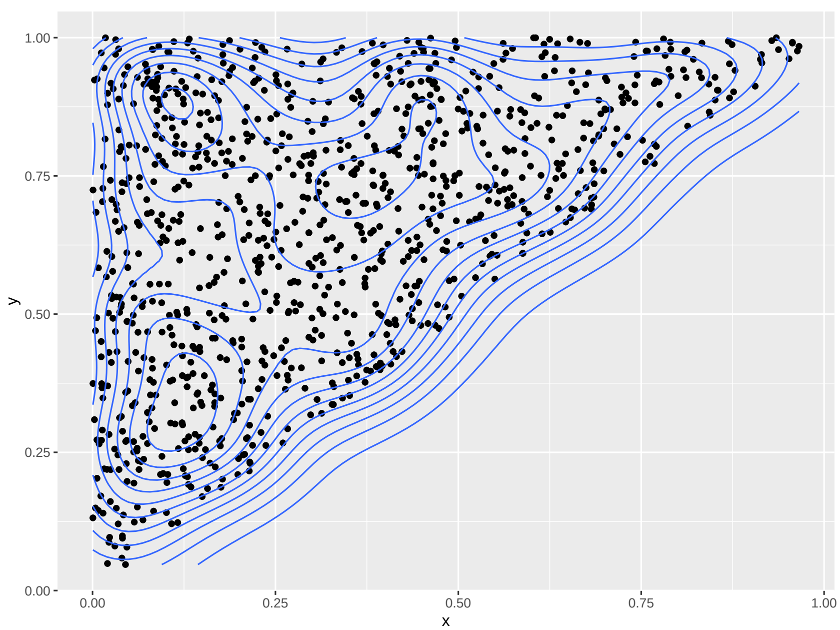

Figure 2